9.1 Harmonic Motion

A bicyclist pedaling past you on the street moves in **linear motion**. Linear motion gets us from one place to This chapter is about another kind of motion called harmonic motion(Figure 9.1B). **Harmonic motion** is motion that repeats over and over. The cycles of day and night and the seasons are caused by Earth's harmonic motion. Other examples of harmonic motion cause your heartbeat and create the sound of music.

VOCABULARY

linear motion - motion that goes from one place to another without repeating.
harmonic motion - motion that repeats in cycles.
cycle - a unit of motion that repeats.
pendulum - a device that swings back and forth due to the force of gravity.

Motion in cycles

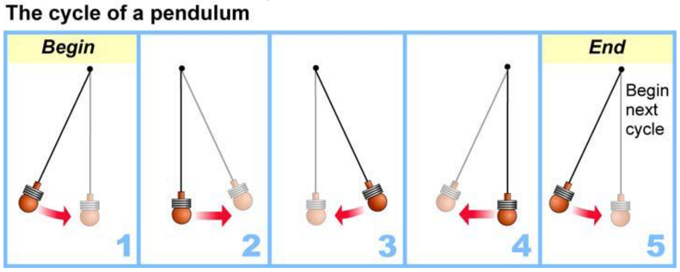
What is a cycle?

To describe harmonic motion we need to learn some new ideas that describe the "over-and-over" repeating action. The first important idea is the **cycle**. One full back-and-forth swing of a child on a swing is one cycle. The motion of the swing is the same cycle, repeated over and over again.

Looking at one cycle

A **pendulum's** cycle is shown in the diagram below. Each box in the diagram is a snapshot of the motion at a different time in the cycle.

The cycle of a pendulum



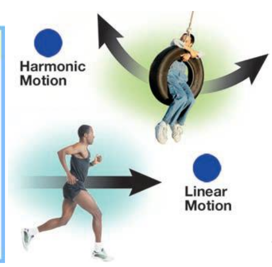


Figure 9.1: *(A) A person on a swing is a good example of harmonic motion. (B) A sprinter is a good example of linear motion.*

The cycle of a pendulum

The cycle starts with (1) the swing from left to center. Next, the cycle continues with (2) center to right, and (3) back from right to center. The cycle ends when the pendulum moves (4) from center to left because this brings the pendulum back to the beginning of the next cycle. Once a cycle is completed, the next cycle begins without any interruption in the motion.

Oscillators

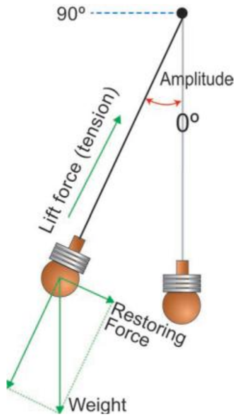
An **oscillator** is a physical system that has repeating cycles. A child on a swing is an oscillator, as is a vibrating guitar string. A wagon on a hill just rolls down and is not an oscillator. What properties determine whether a system will oscillate or not?

Equilibrium

Systems that oscillate always move back and forth around a center or *equilibrium* position. You can think of equilibrium as the system at rest, undisturbed, with zero net force. A wagon on a hill is *not* in equilibrium because the force of gravity is not balanced by another force. A child sitting *motionless* on a swing *is* in equilibrium because the force of gravity is balanced by the tension in the ropes.

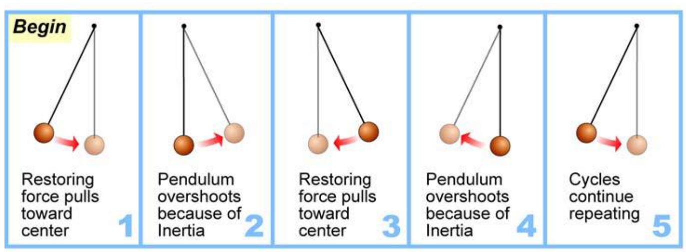
Restoring forces

A **restoring force** is any force that always acts to pull a system back toward equilibrium. If a pendulum is pulled forward, gravity creates a restoring force that pulls it back, toward equilibrium. If the pendulum is moved backward, gravity pulls it forward, back to equilibrium again (Figure 9.2). *Systems with restoring forces are the ones that become oscillators.*



Inertia causes an oscillator to go past equilibrium

Gravity always pulls a pendulum toward equilibrium. Why doesn't the pendulum just stop at equilibrium? Newton's first law of motion explains why. According to the first law, an object in motion tends to stay in motion. The pendulum has inertia that keeps it moving forward. Inertia causes the pendulum to overshoot its equilibrium position every time. The result is harmonic motion.



Frequency and period

A period is the time for one cycle

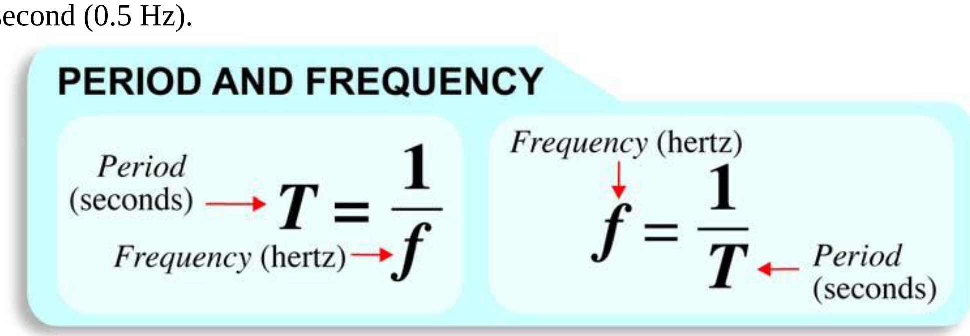
Harmonic motion can be fast or slow, but we don't use speed to tell the difference. This is because the speed of a pendulum constantly changes during its cycle. We use *period* and *frequency* to describe how quickly cycles repeat themselves. The time for one cycle to occur is called a **period**. A clock pendulum with a period of one second will complete one full back and forth swing each second.

Frequency is the number of cycles per second

The **frequency** is the number of complete cycles per second. The unit of one cycle per second is called a **hertz**, abbreviated (Hz). Something that completes ten cycles each second has a frequency of 10 Hz. A guitar string playing the note "A" vibrates back and forth at a frequency of 220 Hz (Figure 9.3). Your heartbeat has a frequency between one-half and two cycles per second (0.5 Hz–2 Hz).

Frequency is the inverse of period

Frequency and period are inversely related. The period is the number of seconds per cycle. The frequency is the number of cycles per second. For example, if the period of a pendulum is 2 seconds, its frequency is 0.5 cycles per second (0.5 Hz).



When to use period or frequency

While both period and frequency tell us the same information, we usually use period when cycles are slower than a few per second. The pendulum you use in the lab has period between 0.9 and 2 seconds. We use frequency when cycles repeat faster. The vibrations that make sound in musical instruments have frequencies between 20 and 20,000 Hz.

Amplitude

Amplitude describes the “size” of a cycle

The **amplitude** describes the “size” of a cycle. Figure 9.4 shows a pendulum with small amplitude and one with a large amplitude. With a moving object like a pendulum, the amplitude is often a distance or angle. With other kinds of oscillators, the amplitude might be voltage or pressure. The amplitude is measured in units appropriate to the kind of harmonic motion being described.

How do you measure amplitude?

The amplitude is the maximum distance the oscillator moves away from its *equilibrium* position. For a pendulum, the equilibrium position is hanging straight down in the center. For the pendulum in Figure 9.5, the amplitude is 20 degrees, because the pendulum moves 20 degrees away from center in either direction.

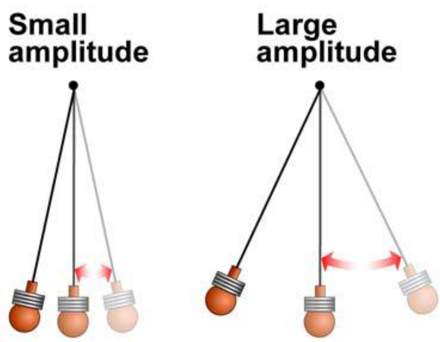


Figure 9.4: *Small amplitude versus large amplitude*

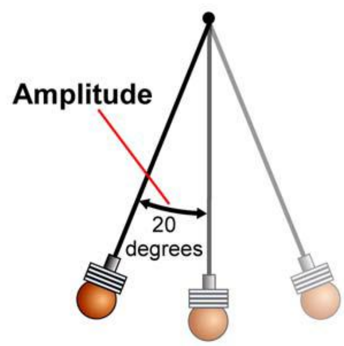


Figure 9.5: *A pendulum with an amplitude of 20 degrees swings 20 degrees away from the center in either direction.*

Damping and friction

Look at the illustration above. Friction slows a pendulum down, just as it slows all motion. That means the amplitude slowly gets reduced until the pendulum is hanging straight down, motionless. We use the word *damping* to describe the gradual loss of amplitude. If you wanted to make a clock with a pendulum, you would have to find a way to keep adding energy to counteract the damping of friction so the clock’s pendulum would work continuously.

A good way to find the amplitude is to measure the distance between the farthest points the motion reaches. The amplitude is half this distance. The amplitude of a water wave is often found this way.

Graphs of Harmonic Motion

Graphing harmonic motionA graph is a good way to show harmonic motion because you can quickly recognize cycles (Figure 9.6). The most common type of graph puts position on the vertical (y) axis and time on the horizontal (x) axis. The graph below shows how the position of a pendulum changes over time. The repeating “wave” on the graph represents the repeating cycles of motion of the pendulum.

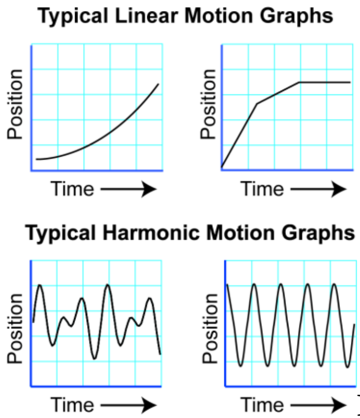
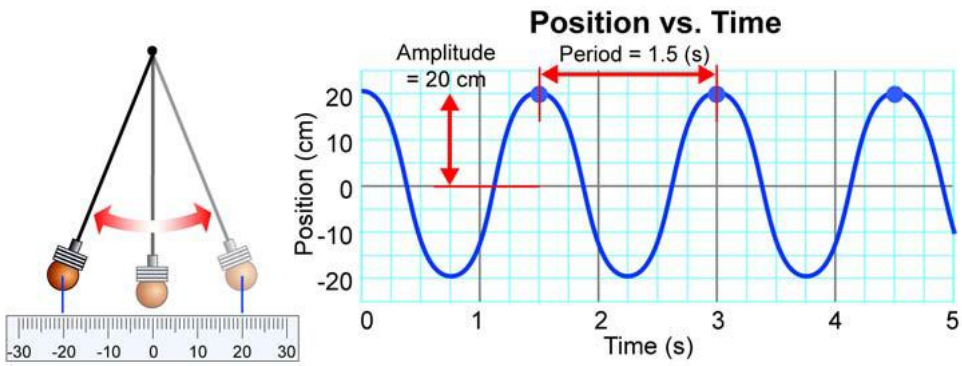


Figure 9.6: *Typical graphs for linear motion (top) and harmonic motion (bottom). Graphs of linear motion do not show cycles. Harmonic motion graphs show repeating cycles.*



Finding the period

This pendulum has a period of 1.5 seconds, so the pattern on the graph repeats every 1.5 seconds. If you were to cut out any piece of the graph and slide it over 1.5 seconds it would line up exactly. You can tell the period is 1.5 seconds because the graph repeats itself every 1.5 seconds.

Showing amplitude on a graph

The amplitude of harmonic motion can also be seen on a graph. The graph above shows that the pendulum swings from +20 centimeters to –20 centimeters and back. Therefore, the amplitude of the pendulum is 20 centimeters.

Using positive and negative positions

Harmonic motion graphs often use positive and negative values to represent motion on either side of a center (equilibrium) position. Zero usually represents the equilibrium point. Notice that zero is placed halfway up the *y*-axis so there is room for both positive and negative values. This graph is in centimeters, but the motion of the pendulum could also have been graphed using the angle measured relative to the center (straight down) position.

Natural frequency and resonance

Natural frequency

A pendulum will have the same period each time you set it moving. Unless you change the pendulum itself (such as changing its length), it will always swing with the same period. The **natural frequency** is the frequency (or period) at which a system naturally oscillates. Every system that oscillates has a natural frequency. Musical instruments use natural frequency. For example, guitar strings are tuned by adjusting their natural frequency to match musical notes.

Changing natural frequency

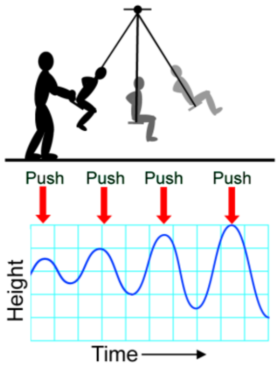
Natural frequency depends on the balance between restoring force and inertia (mass). Any change that affects this balance will also change the natural frequency. The natural frequency of a vibrating guitar string increases when the string gets tighter. Tightening the string increases the force pulling the string back toward equilibrium. Higher force means higher acceleration, and higher natural frequency.

Periodic force

You can get a swing moving by pushing it at the right time every cycle. A force that is repeated over and over is called a **periodic force**. A periodic force has a cycle with an amplitude, frequency, and period, just like an oscillator. *To supply energy to an oscillator you need to use a periodic force.*

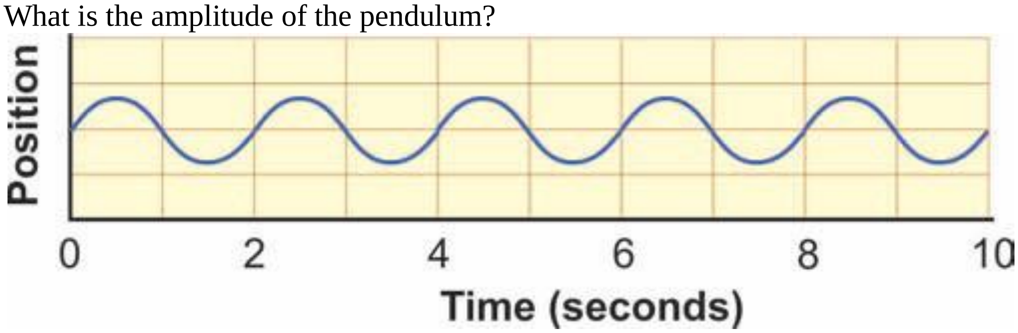
Resonance

Resonance happens when a periodic force has the same frequency as the natural frequency. When this happens, each push adds to the next one and the amplitude of the motion grows. The big amplitude of a swing is an example of resonance. You get a big swing by using many small pushes applied at the right time each cycle, or, in other words, by applying a periodic force (repetitive pushes) at the natural frequency (once each cycle). In resonance, the response can grow very large compared to the strength of the force.



9.1 Section Review

1. Which is the best example of a cycle: a turn of a wheel or a slide down a ski slope?
2. Describe one example of an oscillating system you would find at an amusement park.
3. What is the relationship between period and frequency?
4. Every 6 seconds a pendulum completes 1 cycle. What are the period and frequency of this pendulum?
5. What is the difference between a graph of linear motion and a graph of harmonic motion?
6. A graph of the motion of a pendulum shows that it swings from +5 centimeters to –5 centimeters for each cycle. What is the amplitude of the pendulum?



7. What is the period of the oscillation shown in the diagram above?
8. Figure 9.7 shows a sliding mass on a spring. Assume there is no friction. Will this system oscillate? Explain why or why not.

9. A student makes two oscillators with identical rubber bands that are stretched the same amount (Figure 9.8). One oscillator has a wooden bead in the middle. Which oscillator will have the higher natural frequency? Explain why you think so.

10.Resonance happens when
a. a periodic force is applied at the natural frequency
b. an oscillator has more than one natural frequency
c. a force is periodic and not constant
d. the amplitude of an oscillator grows large over time

VOCABULARY

linear motion - motion that goes from one place to another without repeating.

harmonic motion - motion that repeats in cycles.

cycle - a unit of motion that repeats.

pendulum - a device that swings back and forth due to the force of gravity.

oscillator - a physical system that has repeating cycles.

restoring force - any force that always acts to pull a system back toward equilibrium.

period - the time it takes for each complete cycle.

frequency - how often something repeats, expressed in hertz.

hertz - the unit of frequency. One hertz is one cycle per second.

amplitude - the amount that a cycle moves away from equilibrium.

natural frequency – the frequency at which a system oscillates when disturbed.

periodic force - a repetitive force.

resonance - an exceptionally large amplitude that develops when a periodic force is applied at the natural frequency.